# Physics 1A Review Sheet

Tau Beta Pi - Boelter 6266

## Contents

# 1 Kinematic Equations

## 1.1 Kinematic Equation Variables

$v_0$ - initial velocity of time period being considered
$v_f$ - final velocity of time period being considered
$\Delta x$ - final displacement in chosen positive direction
$t$ - time passed in time period
$a$ - constant acceleration

## 1.2 Kinematic Equations (1-D)

$\Delta x = v_0 t + \frac{1}{2} a t^2$ $\quad v_f = v_0 + at$ $\quad v_f^2 = v_0^2 + 2a\Delta x$ $\quad \frac{v_f + v_0}{2} = \frac{\Delta x}{t}$

The first two equations can be inserted into one another to get the third equation. Therefore, these 4 equations have only 3 independent equations, so only 3 independent variables can be solved for (which means you need to know at least two of the top variables).

## 1.3 Kinematic Equations (2-D)

| | | |
|---|---|---|
| $\Delta l$ - total displacement | $\Delta x$ - x-direction displacement | $\Delta y$ - y-direction displacement |
| $a$ - constant acceleration | $a_x$ - x-direction acceleration | $a_y$ - y-direction acceleration |
| $v_0$ - initial velocity | $v_{0,x}$ - initial x-direction velocity | $v_{0,y}$ - initial y-direction velocity |
| $v_f$ - final velocity | $v_{f,x}$ - final x-direction velocity | $v_{f,y}$ - final y-direction velocity |

All quantities in each row can be related to the total quantity (in the left column) by $\cos(\theta)$ for x and $\sin(\theta)$ for y. For example,

$$\Delta x = \Delta l \cos(\theta) \qquad \Delta y = \Delta l \sin(\theta)$$

# 2 Newton's Second Law

For a constant mass, m, with the total forces on it being $\sum F$, and acceleration a, we have $\sum F = ma$. This equation must be solved for using known force equations and solving for others as needed.

Spring Force - $F_x = -kx$ where k is the spring constant and x is the displacement of spring
Normal Force - $F_N$ No known form, but usually solved with a force balance in this direction
Kinetic Friction Force - $f_k = \mu_k F_n$ where $\mu_k$ is the kinetic coefficient of friction
Static Friction Force - $f_s \leqslant \mu_s F_N$ where $\mu_s$ is the static coefficient of friction; equality when going to slip
Tension - $T$ No known form, must be solved for using Newton's second law equations
Gravity near Earth's surface - $F_g = -mg$ where m is the mass of the object considered and g is $9.81 \frac{m}{s^2}$
Gravity (general) - $F_g = \frac{-GmM}{r^2}$ where $G = 6.67 \times 10^{-11} \frac{N-m^2}{kg^2}$ where m is the mass of the object, M is the mass of the other object, r is the distance between them (force is attractive for both objects)

## 2.1 Uniform Circular Motion

For a mass moving in circular orbit, let $r$ be the radius of the circular path, $a_r$ be the acceleration of the object pointing towards the center of the circle, and $v$ be the instantaneous velocity. Then, we have

$$a = \frac{v^2}{r}$$

This equation also holds true even if v is not constant.

## 3 Conservation of Energy

Spring Potential Energy - $PE_{spring} = \frac{1}{2}kx^2$ Gravitational Potential Energy - $PE_{gravity} = mgh$
Non-conservative work $W_{nc}$ - energy added or taken away from the system by non-conservative forces
Kinetic Energy - $KE = \frac{1}{2}mv^2$ Potential Energy - $PE$ - sum of potential energy terms
Work-energy theorem - $W_{nc} = \Delta KE + \Delta PE$ where the change refers to the change of kinetic and potential terms between the 2 times being considered.

## 4 Conservation of Momentum

Momentum - $p = mv$ describes a quantity such that $\frac{dp}{dt} = F$
Conservation of momentum in linear collisions: $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$ where $m_1$ and $m_2$ are the masses of the objects colliding, $v_1$ and $v_2$ are the velocities before and before the collision, and $v_1'$ and $v_2'$ are the velocities after the collision
Conservation of momentum (general): $m_1\vec{v_1} + m_2\vec{v_2} = m_1\vec{v_1}' + m_2\vec{v_2}'$
Linear 1-D Elastic Collision equation: $v_1 + v_1' = v_2 + v_2'$
Variable Mass Equation: $\sum F = m(t)a(t) - \frac{dm(t)}{dt}v_{rel}(t)$ where m(t) is the instantaneous mass of the system, $v_rel(t)$ is the velocity of the output stream relative to the system, and a(t) is the acceleration

## 5 Center of Mass

Given masses $m_1$,...,$m_n$, the center of mass is calculated as

$$(\sum_{i=1}^{n} m_i)x_{cm} = \sum_{i=1}^{n} m_i x_i = m_1x_1 + ... + m_nx_n \Rightarrow x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

For continuous distributions, we have

$$x_{cm}\int_{sys} dm = \int_{sys} x\,dm \Rightarrow x_{cm} = \frac{\int_{sys} x\,dm}{m_{sys}} = \frac{\int_{sys} x\,dm}{\int_{sys} dm}$$

We can take derivatives of the equation above with respect to time to get useful expressions.

$$(\sum_{i=1}^{n} m_i)v_{cm} = \sum_{i=1}^{n} m_i v_i = m_1v_1 + ... + m_nv_n \Rightarrow v_{cm} = \frac{\sum_{i=1}^{n} m_i v_i}{\sum_{i=1}^{n} m_i}$$

$$(\sum_{i=1}^{n} m_i)a_{cm} = \sum_{i=1}^{n} m_i a_i = m_1a_1 + ... + m_na_n = \sum F \Rightarrow a_{cm} = \frac{\sum_{i=1}^{n} m_i a_i}{\sum_{i=1}^{n} m_i}$$

By the last expression and Newton's third law applying equally in opposite directions, the forces cancel out in the summation for the case of collisions/separations, so we have that the acceleration of the center of mass does not change in collisions.

## 6 Rotations

Moment of Inertia- $I = \int r^2\,dm$ where r is the distance from the axis of rotation
Angle Displaced- $\Delta\theta$ Angular Velocity- $\omega = \frac{d\theta}{dt}$ Angular acceleration- $\alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}$
Torque- $\vec{\tau} = \vec{r} \times \vec{F}$ Angular Momentum- $\vec{L} = \vec{r} \times \vec{p}$ Rotational Kinetic Energy- $KE = \frac{1}{2}I\omega^2$

Equations written for linear variables have analogs for rotations.

$$\sum F = ma \Rightarrow \sum \tau = I\alpha$$

$$KE_{trans} = \frac{1}{2}mv^2 \Rightarrow KE_{rot} = \frac{1}{2}I\omega^2$$
$$p = mv \Rightarrow L = I\omega$$
$$\sum F = \frac{dp}{dt} \Rightarrow \sum \tau = \frac{dL}{dt}$$

Kinetic Energies from both rotational and translational must be considered in calculating a total.

$$KE_{tot} = KE_{trans} + KE_{rot}$$

We also have conservation of angular momentum for systems where the collisions do not result in a torque with a component perpendicular to the plane of spin.

$$I_1\omega_1 + I_2\omega_2 = I_1\omega_1' + I_2\omega_2'$$

Requirements for equilibrium of a system: the net forces and torques around any center must be 0.

$$\sum F = 0 \qquad \sum \tau = 0$$

Requirements for rolling without slipping for objects with constant radii:

$$r\alpha = a_{cm} \qquad r\omega = v_{cm}$$

# 7 Gravitation

Newton's Law of Gravitation: $\vec{F_g} = \frac{-GMm}{r^2}\hat{r}$ where $\hat{r}$ points in direction to the object the force acts on to the other object involved in the attraction.
The gravitational field that a single mass emits is instead given by the equation.

$$\vec{g} = \frac{-GM}{r^2}\hat{r}$$

For continuous distributions, the gravitational field must be integrated over.

$$\vec{g} = \int \frac{-GM}{r^2}\, dm$$

For discrete distributions, the gravitational field is simply the sum of all those acting on the point.

$$\vec{g} = \sum_{i=1}^{n} \vec{g_i} = \vec{g_1} + ... + \vec{g_n}$$