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1 Kinematic Equations

1.1 Kinematic Equation Variables

- $v_0$ - initial velocity of time period being considered
- $t$ - time passed in time period
- $v_f$ - final velocity of time period being considered
- $\Delta x$ - final displacement in chosen positive direction
- $a$ - constant acceleration

1.2 Kinematic Equations (1-D)

\[ \Delta x = v_0 t + \frac{1}{2} at^2 \]
\[ v_f = v_0 + at \]
\[ v_f^2 = v_0^2 + 2a\Delta x \]
\[ \frac{v_f + v_0}{2} = \frac{\Delta x}{t} \]

The first two equations can be inserted into one another to get the third equation. Therefore, these 4 equations have only 3 independent equations, so only 3 independent variables can be solved for (which means you need to know at least two of the top variables).

1.3 Kinematic Equations (2-D)

- $\Delta l$ - total displacement
- $\Delta x$ - x-direction displacement
- $\Delta y$ - y-direction displacement
- $a$ - constant acceleration
- $a_x$ - x-direction acceleration
- $a_y$ - y-direction acceleration
- $v_0$ - initial velocity
- $v_{0,x}$ - initial x-direction velocity
- $v_{0,y}$ - initial y-direction velocity
- $v_f$ - final velocity
- $v_{f,x}$ - final x-direction velocity
- $v_{f,y}$ - final y-direction velocity

All quantities in each row can be related to the total quantity (in the left column) by $\cos(\theta)$ for x and $\sin(\theta)$ for y. For example,

\[ \Delta x = \Delta l \cos(\theta) \quad \Delta y = \Delta l \sin(\theta) \]

2 Newton's Second Law

For a constant mass, m, with the total forces on it being $\sum F$, and acceleration a, we have $\sum F = ma$. This equation must be solved for using known force equations and solving for others as needed.

- Spring Force - $F_x = -kx$ where k is the spring constant and x is the displacement of spring
- Normal Force - $F_N$ No known form, but usually solved with a force balance in this direction
- Kinetic Friction Force - $f_k = \mu_k F_N$ where $\mu_k$ is the kinetic coefficient of friction
- Static Friction Force - $f_s \leq \mu_s F_N$ where $\mu_s$ is the static coefficient of friction; equality when going to slip
- Tension - T No known form, must be solved for using Newton’s second law equations
- Gravity near Earth’s surface - $F_g = -mg$ where m is the mass of the object considered and g is $9.81 \text{m/s}^2$
- Gravity (general) - $F_g = -\frac{GmM}{r^2}$ where $G = 6.67 \times 10^{-11} \text{N}\cdot\text{m}^2/\text{kg}^2$ where m is the mass of the object, M is the mass of the other object, r is the distance between them (force is attractive for both objects)

2.1 Uniform Circular Motion

For a mass moving in circular orbit, let $r$ be the radius of the circular path, $a_r$ be the acceleration of the object pointing towards the center of the circle, and $v$ be the instantaneous velocity. Then, we have

\[ a = \frac{v^2}{r} \]

This equation also holds true even if $v$ is not constant.
3 Conservation of Energy

Spring Potential Energy - \( PE_{spring} = \frac{1}{2} kx^2 \)

Gravitational Potential Energy - \( PE_{gravity} = mgh \)

Non-conservative work \( W_{nc} \) - energy added or taken away from the system by non-conservative forces

Kinetic Energy - \( KE = \frac{1}{2} m v^2 \)

Potential Energy - \( PE \) - sum of potential energy terms

Work-energy theorem - \( \dot{W}_{nc} = \Delta KE + \Delta PE \) where the change refers to the change of kinetic and potential terms between the 2 times being considered.

4 Conservation of Momentum

Momentum - \( p = mv \) describes a quantity such that \( \frac{dp}{dt} = F \)

Conservation of momentum in linear collisions: \( m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \) where \( m_1 \) and \( m_2 \) are the masses of the objects colliding, \( v_1 \) and \( v_2 \) are the velocities before and before the collision, and \( v_1' \) and \( v_2' \) are the velocities after the collision

Conservation of momentum (general): \( m_1 v_1' + m_2 v_2' = m_1 v_1 + m_2 v_2 \)

Linear 1-D Elastic Collision equation: \( v_1 + v_2 = v_1' + v_2' \)

Variable Mass Equation: \( \sum F = m(t) a(t) = \frac{dm(t)}{dt} v_{rel}(t) \) where \( m(t) \) is the instantaneous mass of the system, \( v_{rel}(t) \) is the velocity of the output stream relative to the system, and \( a(t) \) is the acceleration

5 Center of Mass

Given masses \( m_1, \ldots, m_n \), the center of mass is calculated as

\[
(\sum_{i=1}^{n} m_i)x_{cm} = \sum_{i=1}^{n} m_i x_i = m_1 x_1 + \ldots + m_n x_n \Rightarrow x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}
\]

For continuous distributions, we have

\[
x_{cm} \int_{sys} dm = \int_{sys} x \ dm \Rightarrow x_{cm} = \frac{\int_{sys} x \ dm}{m_{sys}} = \frac{\int_{sys} x \ dm}{\int_{sys} dm}
\]

We can take derivatives of the equation above with respect to time to get useful expressions.

\[
(\sum_{i=1}^{n} m_i)v_{cm} = \sum_{i=1}^{n} m_i v_i = m_1 v_1 + \ldots + m_n v_n \Rightarrow v_{cm} = \frac{\sum_{i=1}^{n} m_i v_i}{\sum_{i=1}^{n} m_i}
\]

\[
(\sum_{i=1}^{n} m_i)a_{cm} = \sum_{i=1}^{n} m_i a_i = m_1 a_1 + \ldots + m_n a_n = \sum F \Rightarrow a_{cm} = \frac{\sum_{i=1}^{n} m_i a_i}{\sum_{i=1}^{n} m_i}
\]

By the last expression and Newton’s third law applying equally in opposite directions, the forces cancel out in the summation for the case of collisions/separations, so we have that the acceleration of the center of mass does not change in collisions.

6 Rotations

Moment of Inertia- \( I = \int r^2 \ dm \) where \( r \) is the distance from the axis of rotation

Angle Displaced- \( \Delta \theta \) \quad Angular Velocity- \( \omega = \frac{d\theta}{dt} \) \quad Angular acceleration- \( \alpha = \frac{d^2 \theta}{dt^2} = \frac{d\omega}{dt} \)

Torque- \( \tau = \vec{r} \times \vec{F} \) \quad Angular Momentum- \( \vec{L} = \vec{r} \times \vec{p} \) \quad Rotational Kinetic Energy- \( KE = \frac{1}{2} I \omega^2 \)

Equations written for linear variables have analogs for rotations.

\[
\sum F = ma \Rightarrow \sum \tau = I \alpha
\]
\[ KE_{\text{trans}} = \frac{1}{2}mv^2 \Rightarrow KE_{\text{rot}} = \frac{1}{2}I\omega^2 \]

\[ p = mv \Rightarrow L = I\omega \]

\[ \sum F = \frac{dp}{dt} \Rightarrow \sum \tau = \frac{dL}{dt} \]

Kinetic Energies from both rotational and translational must be considered in calculating a total.

\[ KE_{\text{tot}} = KE_{\text{trans}} + KE_{\text{rot}} \]

We also have conservation of angular momentum for systems where the collisions do not result in a torque with a component perpendicular to the plane of spin.

\[ I_1\omega_1 + I_2\omega_2 = I_1\omega'_1 + I_2\omega'_2 \]

Requirements for equilibrium of a system: the net forces and torques around any center must be 0.

\[ \sum F = 0 \quad \sum \tau = 0 \]

Requirements for rolling without slipping for objects with constant radii:

\[ r\alpha = a_{cm} \quad r\omega = v_{cm} \]

### 7 Gravitation

Newton’s Law of Gravitation: \( \vec{F}_g = -\frac{GMm}{r^2}\hat{r} \) where \( \hat{r} \) points in direction to the object the force acts on to the other object involved in the attraction.

The gravitational field that a single mass emits is instead given by the equation.

\[ \vec{g} = -\frac{GM}{r^2}\hat{r} \]

For continuous distributions, the gravitational field must be integrated over.

\[ \vec{g} = \int -\frac{GM}{r^2} \; dm \]

For discrete distributions, the gravitational field is simply the sum of all those acting on the point.

\[ \vec{g} = \sum_{i=1}^{n} \vec{g}_i = \vec{g}_1 + \ldots + \vec{g}_n \]